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INTELLIGENT COMPOSITE STRUCTURES: GENERAL THEORY AND APPLICATIONS

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Abstract—Basic aspects of a newly suggested theory of intelligent composite structures are formulated and discussed. Governing equations describing the behavior of an elastic composite structure incorporating sensors and actuators are developed, and the basic optimization problems are formulated. The theory is illustrated by three examples of a practical interest, which illustrate three major sources of control for an intelligent structure, namely, residual stresses, material properties, and geometry of the structure. In the first example, the optimal residual strain in an actuator which provides the minimal deflection of a cantilevered beam is derived. The second example is concerned with the optimal control of a Winkler foundation rigidity in the problem of vibrations damping for a simply supported beam under dynamic loading. In the third example, for a given residual strain, the optimal length of the actuator that minimizes the beam deflection is obtained, and the dependence of the optimal length on the intensity of residual strain is calculated. Copyright © 1996 Elsevier Science Ltd.

1. INTRODUCTION

In order to formulate the problems in the theory of intelligent structures, we begin with a brief discussion of some relevant aspects of the optimal control theory.

Problems in optimal control and optimal design of flexible structures attracted increasing attention in the past two decades. Despite a large number of applications, investigations were concerned mainly with the dynamics and control of robots, see e.g., Brady *et al.* (1983), Dagli and Kusiak (1985), Khorasani (1992), and aerospace structures, see e.g., Anthony and Wie (1990), Balas (1979), Bar-Kana *et al.* (1983), Meirovitch (1990), Sharon (1992).

The following two basic types of control can be distinguished: (i) passive control with the use of auxiliary non-adaptable elastic springs, viscoelastic dampers, and dynamic absorbers, see e.g., Nashif *et al.* (1985); and (ii) active control, when the undesirable oscillations are counteracted by auxiliary adaptable mechanisms, for example electromagnetic, electromechanical, electrorheological, pneumatical actuators, or actuators using the shape memory effect, see, e.g., Cho and Heirick (1985), Gandhi and Thompson (1989), Kojima *et al.* (1986), Rogers (1992), Rogers and Wallace (1994), Tani (1992).

As a rule, passive control systems do not require special equipment for measurements, whereas active control systems contain sensors as a necessary part.

Under an intelligent composite structure we mean a structure with sensors and actuators which is actively controlled, and which performs a required motion that is optimal in a class of admissible motions.

Comparing this definition with the definition of an active optimal control system, we can see that they are pretty close. One could expect that the theories of active control and intelligent structures should be quite similar. However, there are two important differences between them. The theory of active control systems draws attention to constructing optimal control, it deals with analytical and numerical methods of optimization, and it is concerned with presently existing technical devices and materials.

As distinguished from that, the newly suggested theory of intelligent structures deals mainly with the ultimate features of structures and it is concerned with optimal design of controllable systems, where controllers ensure minima of some cost functionals. It is assumed in the framework of this theory that all the materials and devices with required properties can be designed and fabricated (either now or in the future), while the objective of the theory is to determine limiting properties of a structure which incorporates these (either real or hypothetical) materials and devices.

The theory of intelligent structures may employ mathematical methods similar to the methods of the optimal control theory, but it is essentially focused on the dependence of the cost functional on the system parameters and on the optimal design of a system which provides the optimal (with respect to a cost functional) properties of an active control. For example, in the simplest problem of active damping of vibrations of a cantilevered beam, see e.g., Lee *et al.* (1989), Su and Tadjbakhsh (1991), the problem of optimal control is to find the signals applied to actuators which minimize the deflection, whereas the problem in the theory of intelligent structures is to choose the properties of actuators and their distribution that ensure the minimal deflection under assumption that all the actuators work in their optimal regimes.

In order to estimate structures in progress, it is important from the engineering standpoint to predict their ultimate (limiting) features bearing in mind that any changes in properties (in an appropriate range) of the main material of a structure, as well as of sensors and actuators, are admissible. This allows us to decide whether properties of the presently existing materials and devices are sufficient for the structure, or new materials and equipments are required for the project.

The exposition is as follows: in Section 2, we derive the governing equations for an intelligent structure which incorporates an elastic solid, sensors, and actuators, and we formulate optimization problems. Sections 3–5 are concerned with the analysis of applied examples which illustrate three basic classes of optimization problems. In Section 3, we deal with a static problem of optimization for a cantilevered beam under the action of a transverse force applied to the free end. Section 4 is concerned with the dynamic problem of vibration damping for a simply supported beam lying on a Winkler foundation with controllable material properties. In Section 5, we consider the bending of a simply supported beam with an actuator under a static load.

2. FORMULATION OF GOVERNING EQUATIONS AND OPTIMIZATION PROBLEMS

Let us consider an inhomogeneous intelligent structure consisting of an elastic solid and a set of sensors and actuators. Generally, both the main solid and auxiliary devices are assumed to be made of composite materials. The structure is in its natural state and occupies a connected domain Ω with a piece-wise smooth boundary Γ. At the initial moment $t = 0$, external forces are applied to the body. The load consists of body forces B and surface tractions b. The surface forces b are applied to a part Γ_b of the boundary Γ. The other part of the boundary, $\Gamma_u = \Gamma\backslash\Gamma_b$, is assumed to be fixed.

Points of Ω refer to coordinates $x = \{x_i\}$, $i = 1, 2, 3$. Denote by $u(t, x)$ the displacement vector and

$$\varepsilon(t, x) = \tfrac{1}{2}[\nabla u(t, x) + \nabla u^T(t, x)] \tag{1}$$

the infinitesimal strain tensor, where ∇ is the gradient operator, and T stands for transpose.

The stress tensor, σ, satisfies the equation of motion

$$\rho\frac{\partial^2 u}{\partial t^2} = \nabla\cdot\sigma + \rho B \tag{2}$$

where ρ is the mass density, and the dot denotes the inner product.

Equations (1) and (2) should be fulfilled both for controllable (with the use of actuators), and for uncontrollable motions. The constitutive equations depend essentially on the presence (or absence) of sensors and actuators.

In the absence of actuators, we treat the material as anisotropic elastic composite. Accordingly, the stress σ_m can be expressed in terms of the strains ε by generalized Hooke's law

$$\sigma_m = C(x) \cdot \varepsilon. \tag{3}$$

Here C is a fourth-rank tensor of elastic moduli, see e.g., Kalamkarov (1992). To take into account the presence of sensors and a structure of the composite material, we assume that the main material is inhomogeneous which is modeled by the dependence $C = C(x)$.

In the presence of actuators, the material is modeled as a two-phase blend of the main material (3) and distributed actuators. The latter means that the number of actuators is rather large, and we can assume that in any small (in the sense of continuous mechanics) domain with volume V, the actuators occupy a subdomain with volume V_a. Dividing V_a by V we define volume density of actuators n_a. As usual in mechanics of blends, we suppose that the volume density coincides with the surface density.

We treat actuators also as anisotropic composite elastic solids with tensor of elastic moduli $C_a(x)$. Similarly to eqn (3) we write

$$\sigma_a = C_a(x) \cdot (\varepsilon - \varepsilon_a), \tag{4}$$

where ε_a is the tensor of residual strains in actuators produced under the action of control signals.

The total stress in the material equals the sum of stresses in the main material and the actuators

$$\sigma = (1 - n_a)\sigma_m + n_a \sigma_a. \tag{5}$$

Substitution of expression (3) and (4) into eqn (5) yield

$$\sigma = [(1 - n_a)C + n_a C_a] \cdot \varepsilon - n_a C_a \cdot \varepsilon_a. \tag{6}$$

Equation (6) demonstrates three sources of control for an intelligent composite structure. First, we can choose the tensor ε_a as a function of time t and spatial coordinates x_i, whereas other parameters are prescribed. In this case the material characteristics do not change, and the motion is controlled only by residual strains in actuators. This kind of control occurs, for example, when electrical signals are applied to piezoelectric elastic actuators, see e.g., Lee and Moon (1989, 1990), Tzou and Gadre (1989), or when thermal loads are applied to shape memory alloys, see e.g., Rogers (1992).

Second, we can choose the tensor C_a as a function of time t and spatial coordinates x_i, whereas other parameters remain prescribed. From the physical standpoint, this means that we vary only elastic properties of material, without changing other characteristics. This kind of control occurs in electro-mechanical systems where some sub-systems are turned on and off under the action of electric signal, or in merely mechanical systems in physical fields (temperature, humidity, radiation, etc.), as well as in structures with electrorheological actuators, see e.g., Gandhi and Thompson (1989).

Third, we can choose parameter n_a as a function of spatial coordinates x_i, whereas other parameters are prescribed. This implies the problem of optimal design for a deformable solid with actuators. Unlike the standard problems of optimal design, we seek here not a shape of the body, but optimal (in some sense) spatial distribution of actuators.

Evidently, these sources of control can be combined. As a result, we formulate the basic problem in the theory of intelligent structures: to find optimal mechanical properties of the main composite material and optimal spatial distribution of sensors and actuators which ensure (under the optimal control of the motion by actuators with the use of information from sensors) the required performance.

The theory can be generalized by accounting for the nonlinear response of both the main material and actuators. The above three major sources of control: residual stresses, material properties, and geometry, remain unchanged.

For given control parameters, eqns (1), (2), and (6) together with the boundary conditions

$$u|_{\Gamma_u} = 0, \quad n \cdot \sigma|_{\Gamma_b} = b \tag{7}$$

determine the behavior of an intelligent composite structure. The problem consists in establishing such control parameters which provide an optimal performance in a class of admissible motions.

As usual in the theory of optimal control, we should formulate (i) a class of admissible motions; (ii) a functional which determines the optimal properties; and (iii) an information which is available to construct the control.

The first and the second issues are more or less standard, and we employ in our study the same restrictions, and the same functionals as those used in the optimal control theory for oscillations in elastic systems, see e.g., Tzafestas (1982).

The third issue is much more interesting. Two different cases should be distinguished. In the first case, external forces and the material properties are assumed to be known. Optimal control is assumed to be a function of time and spatial coordinates, and is not related to observations at all. Some engineering problems of this type are studied later in the present paper.

In the other case, external loads are not known precisely, and optimal control is constructed as a function of observing parameters (sensors' data). It is assumed that several sensors are installed in the structure, and control is formed as a response to their signals. Here an additional class of optimization problems arises: to determine the necessary number of sensors and their optimal location. Moreover, as a rule, information of sensors is incomplete, and it contains random errors. In systems with sensors and actuators, transmission of signals from sensors to actuators, as well as inertia of actuators, cause a delay in control which can lead to the system instability, see e.g., Drozdov and Kolmanovskii (1994). We suggest using viscoelastic materials to prevent the instability caused by the delays in information and response, as well as by random errors in observations. The problems of optimal design of sensors under conditions of uncertainty, and the stability analysis for intelligent viscoelastic structures will be the subject of our future study.

3. OPTIMAL DESIGN OF RESIDUAL STRAINS

Let us consider a cantilevered beam with length l and rectangular cross-section of a unit width. The beam consists of two perfectly bonded layers, see Fig. 1. The upper layer is made of a linear elastic material. It has thickness h and the Young modulus E. The lower layer is made of a piezoelectric elastic material with thickness h_a and the Young modulus E_a. The subscript a means that the lower layer is employed as an actuator. We assume that electric potentials applied to the actuator surfaces produce a residual strain ε_a in the lower

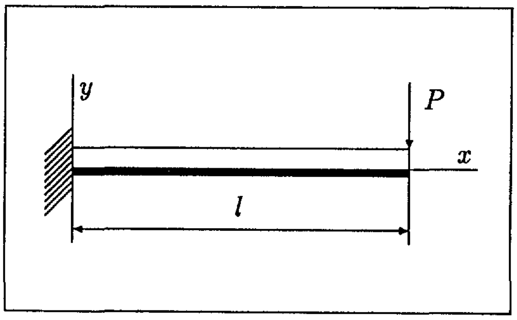

Fig. 1. An intelligent, cantilevered beam with an actuator.

layer. This strain depends only on time, $\varepsilon_a = \varepsilon_a(t)$, and it can be controlled to provide an optimal conduct of the whole structure.

Structures consisting of two (or more) linked layers where one of the layers is employed as an actuator, were considered in a number of investigations, see e.g., Bailey and Hubbard (1985), Barbone and Braga (1992), Lee and Moon (1989), Lee *et al.* (1989), Natori *et al.* (1988), Qiu *et al.* (1994) Tzou and Gadre (1989). These intelligent structures are used in precision optic elements and machine parts such as reflectors and guidelines, where even small bending deformations caused by external forces or variations of temperature can have significant influence on their performance.

The governing equations for composite plates and shells with some active layers were derived, e.g., by Lee and Moon (1989, 1990), Lee *et al.* (1989), Tzou and Gadre (1989). These equations are rather complicated due to their generality. For our needs, we deduce the governing equations for a structure shown in Fig. 1 independently.

Introduce coordinates x and y directed as shown in Fig. 1. Axis x coincides with the interface between the upper and lower layers. Denote by $u(x)$ the displacement in x direction, and by $w(x)$ the beam deflection on the interface.

We assume that (i) the beam displacements are small, and the nonlinear terms in the expressions for the curvature can be neglected; (ii) the hypothesis regarding plane sections at bending holds. The non-zero components u_x and u_y of the displacement field are calculated as follows:

$$u_x = u(x)+yw'(x), \quad u_y = -w(x), \tag{8}$$

where prime denotes the differentiation with respect to x.

According to eqn (8), the only non-zero component of the strain tensor equals

$$\varepsilon_{xx} = u'(x)+yw''(x). \tag{9}$$

Neglecting the Poisson's effect, we assume that the only non-zero component of the stress tensor is σ_{xx}. The stress σ_{xx} is related to the strain ε_{xx} by Hooke's law, cf. eqns (3) and (4),

$$\begin{aligned} \sigma_{xx} &= E\varepsilon_{xx} && \text{in upper layer.} \\ \sigma_{xx} &= E_a(\varepsilon_{xx}-\varepsilon_a) && \text{in lower layer.} \end{aligned} \tag{10}$$

Substitution of expression (9) into eqn (10) yields

$$\begin{aligned} \sigma_{xx} &= E(u'+yw'') && (0 \leqslant y \leqslant h), \\ \sigma_{xx} &= E_a(u'+yw''-\varepsilon_a) && (-h_a \leqslant y < 0). \end{aligned} \tag{11}$$

Let us substitute expression (11) into the formula for the longitudinal force

$$N = \int_{-h_a}^{h} \sigma_{xx}\,\mathrm{d}y.$$

After simple algebra we obtain

$$N = (Eh+E_ah_a)u'+\tfrac{1}{2}(Eh^2-E_ah_a^2)w''-E_ah_a\varepsilon_a. \tag{12}$$

It follows from the equilibrium equations that $N = 0$. This equality together with expression (12) implies that

$$u' = \frac{1}{Eh+E_a h_a}[E_a h_a \varepsilon_a - \tfrac{1}{2}(Eh^2 - E_a h_a^2)w'']. \tag{13}$$

We now substitute expression (11) into the formula for the bending moment

$$M = \int_{-h_a}^{h} \sigma_{xx} y \, \mathrm{d}y$$

and find

$$M = \tfrac{1}{2}(Eh^2 - E_a h_a^2)u' + \tfrac{1}{3}(Eh^3 + E_a h_a^3)w'' + \tfrac{1}{2}E_a h_a^2 \, \varepsilon_a.$$

Substitution of expression (13) into this equality yields

$$M = \frac{1}{2(Eh+E_a h_a)}\{EE_a hh_a(h+h_a)\varepsilon_a + \tfrac{1}{6}[(Eh^2 - E_a h_a^2)^2 + 4EE_a hh_a(h+h_a)^2]w''\}. \tag{14}$$

For $\varepsilon_a = 0$, and $h_a = 0$, eqn (14) implies the well-known formula

$$M = \frac{Eh^3}{12}w''.$$

Let us suppose now that a time-independent transverse force P is applied to the free end of the cantilevered beam. It follows from the equilibrium equations that

$$M(x) = P(l-x).$$

Substitution of expression (14) into this equality yields

$$Aw'' = P(l-x) - B\varepsilon_a, \tag{15}$$

where

$$A = \frac{(Eh^2 - E_a h_a^2)^2 + 4EE_a hh_a(h+h_a)^2}{12(Eh+E_a h_a)}, \quad B = \frac{EE_a hh_a(h+h_a)}{2(Eh+E_a h_a)}. \tag{16}$$

Integrating eqn (15) with boundary conditions

$$w(0) = 0, \quad w'(0) = 0,$$

we obtain

$$w(x) = \frac{x^2}{2A}\left[P\left(l - \frac{x}{3}\right) - B\varepsilon_a\right], \quad w'(x) = \frac{x}{2A}[2(Pl - B\varepsilon_a) - Px]. \tag{17}$$

Let us consider the following optimization problem: find such a value ε_{a*} of strain ε_a, which minimizes the maximal deflection of the beam

$$\max_{0 \leqslant x \leqslant l} |w(x)|.$$

To solve this problem we first fix ε_a and calculate $\max_x |w(x)|$ for this strain, and afterwards, minimize the obtained value with respect to ε_a.

Point ξ where function $w(x)$ reaches extremum, either is located at the free end, $\xi = l$, or coincides with a root x_* of the derivative $w'(x)$. It follows from eqn (17) that

$$x_* = \frac{2}{P}(Pl - B\varepsilon_a). \tag{18}$$

Evidently, we are interested in the roots which belong to the interval $(0, l)$. This together with formula (18) yields

$$\frac{Pl}{2B} < \varepsilon_a < \frac{Pl}{B}. \tag{19}$$

For other values of strain ε_a, the maximal deflection takes place at the free end of the beam, and, according to eqn (17), it equals $|w(l)| = G_1(\varepsilon_a)$, where

$$G_1(\varepsilon_a) = \frac{Pl^3}{3A}\left|1 - \frac{3B\varepsilon_a}{2Pl}\right|. \tag{20}$$

Suppose that inequality (19) holds. Substitution of expression (18) into eqn (17) implies that $|w(x_*)| = G_2(\varepsilon_a)$, where

$$G_2(\varepsilon_a) = \frac{2Pl^3}{3A}\left(1 - \frac{B\varepsilon_a}{Pl}\right)^3. \tag{21}$$

Plots of functions $G_1(\varepsilon_a)$ and $G_2(\varepsilon_a)$ are presented in Fig. 2.

Evidently, the minimal deflection equals the minimum of function

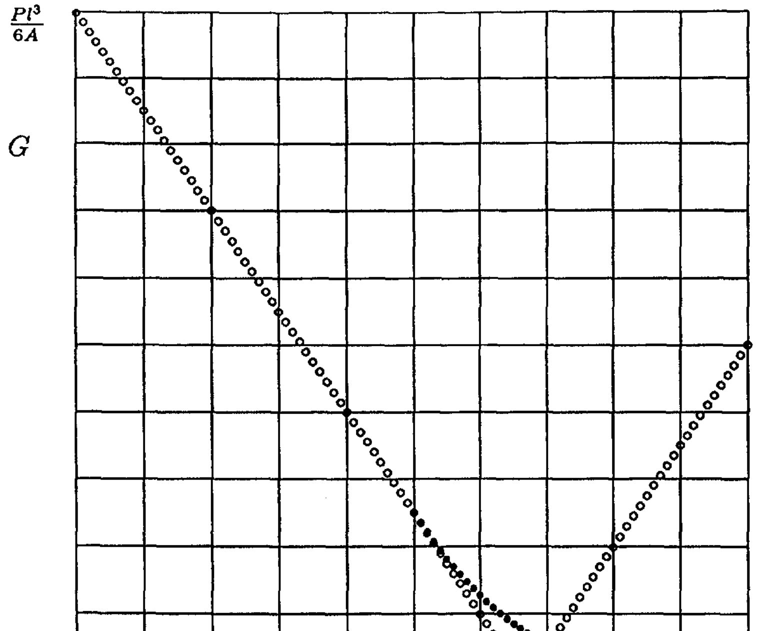

Fig. 2. Dependences $G_1(\varepsilon_a)$ (unfilled circles) and $G_2(\varepsilon_a)$ (filled circles). Function $G(\varepsilon_a) = \max\{G_1(\varepsilon_a), G_2(\varepsilon_a)\}$ reaches its minimal value at point ε_{a*}.

$$G(\varepsilon_a) = \max\{G_1(\varepsilon_a), \quad G_2(\varepsilon_a)\}.$$

This minimum is reached at point ε_{a*} which coincides with the root of the equation

$$G_1(\varepsilon_a) = G_2(\varepsilon_a), \tag{22}$$

located in the interval $[Pl/2B, Pl/B]$. Substitution of expressions (20) and (21) into eqn (22) yields

$$\frac{3B\varepsilon_a}{2Pl} - 1 = 2\left(1 - \frac{B\varepsilon_a}{Pl}\right)^3.$$

Introducing the notation $\varepsilon_a = (Pl/B)z$, we can rewrite this equation as follows:

$$3z - 2 = 4(1-z)^3. \tag{23}$$

The numerical calculation shows that the only root of eqn (23) within the interval $[\frac{1}{2}, 1]$ equals 0.702. Hence the optimal strain in the actuator, which minimizes the beam deflection can be calculated according to the formula

$$\varepsilon_{a*} = 0.702\frac{Pl}{B} = 1.404\frac{Pl(Eh+E_a h_a)}{EE_a hh_a(h+h_a)}.$$

The maximal deflection of the beam equals

$$\max_x |w(x)| = 0.018\frac{Pl^3}{A}. \tag{24}$$

Expression (24) allows us to predict the limiting properties of the actuator. It follows from eqn (17) that in the absence of control, $\varepsilon_a = 0$, the maximal deflection of the beam equals

$$\max_x |w(x)| = 0.5\frac{Pl^3}{A}. \tag{25}$$

Comparison of expressions (24) and (25) shows that the optimal control of the actuator allows the maximal deflection to be reduced by 28.3 times.

Let us now generalize the above problem and consider the beam under the action of an arbitrary distributed transverse load with a moment $\mu(x) \geqslant 0$. In this case, eqn (15) can be presented as follows.

$$Aw''(x) = \mu(x) - B\varepsilon_a. \tag{26}$$

Introduce the notations

$$M_1(x) = \int_0^x \mu(\xi)\,d\xi, \quad M_2(x) = \int_0^x (x-\xi)\mu(\xi)\,d\xi.$$

Similarly to eqn (17) we find from eqn (26)

$$w(x) = \frac{1}{A}\left[M_2(x) - \tfrac{1}{2}B\varepsilon_a x^2\right], \quad w'(x) = \frac{1}{A}[M_1(x) - B\varepsilon_a x]. \tag{27}$$

It follows from eqn (27) that for any $\varepsilon_a > 0$, function $|w(x)|$ has only one point of minimum $x_*(\varepsilon_a)$ which coincides with the root of the equation

$$\mathscr{F}_1 \equiv M_1(x_*) - B\varepsilon_a x_* = 0, \tag{28}$$

which belongs to the interval $(0, l)$. By repeating the above calculations, we obtain the following equation for determining the optimal strain ε_{a*} in the actuator:

$$\mathscr{F}_2 \equiv M_2(x_*) - M_2(l) + \tfrac{1}{2}B\varepsilon_a(l^2 - x_*^2) = 0. \tag{29}$$

The strain ε_{a*} is found numerically from eqn (29). Afterwards, substitution of the obtained value into eqn (27) implies an expression for the optimal deflection of the beam.

It is of interest to analyze robustness of the above algorithm. For this purpose, we assume that

$$\mu(x) = P(l-x) + \eta\tilde{\mu}(x),$$

where $\tilde{\mu}(x)$ is a bounded function, and η is a small parameter. In this case,

$$M_1(x,\eta) = Px\left(l - \frac{x}{2}\right) + \eta\tilde{M}_1(x), \quad M_2(x,\eta) = \frac{Px^2}{2}\left(l - \frac{x}{3}\right) + \eta\tilde{M}_2(x), \tag{30}$$

where $\tilde{M}_1(x)$ and $\tilde{M}_2(x)$ are continuous functions bounded on $[0, l]$. We seek solutions of eqns (28) and (29) in the form of the Taylor series

$$x_* = x_*^{(0)} + \eta x_*^{(1)} + \cdots, \quad \varepsilon_a = \varepsilon_a^{(0)} + \eta\varepsilon_a^{(1)} + \cdots. \tag{31}$$

Substitution of expressions (31) into eqns (28) and (29) implies that

$$\mathscr{F}_1(x_*^{(0)}, \varepsilon_a^{(1)}, 0) = 0, \quad \mathscr{F}_2(x_*^{(0)}, \varepsilon_a^{(1)}, 0) = 0, \tag{32}$$

and

$$\begin{aligned} \frac{\partial\mathscr{F}_1}{\partial x_*}x_*^{(1)} + \frac{\partial\mathscr{F}_1}{\partial\varepsilon_a}\varepsilon_a^{(1)} &= -\frac{\partial\mathscr{F}_1}{\partial\eta}, \\ \frac{\partial\mathscr{F}_2}{\partial x_*}x_*^{(1)} + \frac{\partial\mathscr{F}_2}{\partial\varepsilon_a}\varepsilon_a^{(1)} &= -\frac{\partial\mathscr{F}_2}{\partial\eta}, \end{aligned} \tag{33}$$

where the derivatives are calculated at the point $(x_*^{(0)}, \varepsilon_a^{(0)}, 0)$.

Equations (32) yield solution (18) to the optimization problem for a cantilevered beam under the action of a transverse force P applied to the free end. By using this solution, we find from eqns (33)

$$\varepsilon_a^{(1)} = \frac{2}{B}\frac{\tilde{M}_2(l) - \tilde{M}_2(x_*^{(0)})}{l^2 - x_*^{(0)2}}. \tag{34}$$

It follows from eqn (18) that $x_*^{(0)} = 2l(1-z)$. Substitution of this expression into eqn (34) implies that

$$\varepsilon_a^{(1)} = \frac{2}{Bl^2[1-4(1-z)^2]}\left\{\int_0^l (l-\xi)\tilde{\mu}(\xi)\mathrm{d}\xi - \int_0^{2l(1-z)} [2l(1-z)-\xi]\tilde{\mu}(\xi)\,\mathrm{d}\xi\right\}. \tag{35}$$

Let us assume that

$$\tilde{\mu}_0 = \frac{1}{l}\int_0^l \tilde{\mu}(\xi)\,\mathrm{d}\xi \tag{36}$$

is fixed. It is easy to show that under condition (36) the right-hand side of eqn (35) reaches maximum for

$$\tilde{\mu}(\xi) = \tilde{\mu}_0 l\delta(\xi - 2l(1-z)+0), \tag{37}$$

where $\delta(x)$ is the Dirac delta-function. In this case, the first integral in eqn (35) is maximal, and the second integral vanishes. Substitution of expression (37) into eqn (35) yields

$$\mathrm{e}_a^{(1)} = \frac{2\tilde{\mu}_0}{B[1+2(1-z)]} = \frac{4.950\tilde{\mu}_0}{B}. \tag{38}$$

Formula (38) serves to evaluate the effect of imperfections and uncertainties in external loads on the optimal residual strain. For example, it implies that the growth of B leads to the decrease of the effect of imperfections.

4. OPTIMAL DESIGN OF MATERIAL PROPERTIES

Let us consider a simply supported elastic beam lying on an intelligent, Winkler elastic foundation. The beam has length l, cross-sectional area S, moment of inertia I, and mass density ρ. These parameters are assumed to be independent of the longitudinal coordinate x.

At instant $t = 0$, a distributed transverse load $q(x)$ is applied to the beam. Under the standard assumptions of the technical theory of bending, the beam deflection, $w(t, x)$, obeys the following equation:

$$\rho S\frac{\partial^2 w}{\partial t^2}(t,x) + EI\frac{\partial^4 w}{\partial x^4}(t,x) + g(t,x) - q(x) = 0 \tag{39}$$

with the boundary conditions

$$w(t,0)+w(t,l) = 0, \quad \frac{\partial^2 w}{\partial x^2}(t,0) = \frac{\partial^2 w}{\partial x^2}(t,l) = 0, \tag{40}$$

and the initial conditions

$$w(0,x) = 0, \quad \frac{\partial w}{\partial t}(0,x) = 0. \tag{41}$$

Here $g(t, x)$ is the foundation response at point x at moment t.

We assume that this response is controllable due to changes in the foundation rigidity. Namely, we assume that

$$g(t,s) = c(t)w(t,x). \tag{42}$$

Here $c(t)$ is a piece-wise continuous function taking its values from the interval

$$c_1 \leqslant c(t) \leqslant c_2, \tag{43}$$

where c_1 and c_2 are given positive constants, $c_1 \leqslant c_2$.

From the engineering point of view, such a foundation can be designed as a Winkler foundation including two kinds of springs. The springs of the first kind are non-controllable, their deposit to the total rigidity is characterized by term c_1. The springs of the other kind are controllable. They can be "turned on" and "turned off" by a control electrical signal. When the signal is absent all these springs are turned off, and no additional response arises in the foundation. When the signal is present, some springs are turned on, and their number is proportional to the signal intensity. The maximal response means that all these springs work, and their contribution into the total rigidity increases its value up to c_2.

Substitution of expressions (42) and (43) into eqn (39) yields

$$\rho S \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^2 w}{\partial x^4} + cw = q. \tag{44}$$

Let w_0 be the characteristic deflection of the beam. Introduce the dimensionless variable and parameters

$$w_* = \frac{w}{w_0}, \quad x_* = \frac{x}{l}, \quad t_* = \frac{t}{T_0},$$

$$q_* = \frac{ql^4}{EIw_0}, \quad z = \frac{cl^4}{EI}, \quad T_0 = \sqrt{\frac{\rho S^4}{EI}}.$$

In this notation eqn (44) can be written as follows (for simplicity asterisks are omitted):

$$\frac{\partial^2 w}{\partial t^2} + \frac{\partial^4 w}{\partial x^4} + z(t)w = q. \tag{45}$$

Our objective is to find the optimal control of the foundation rigidity $z^0(t)$, which minimizes the maximal deflection of the beam

$$\max_{0 \leqslant t \leqslant \infty,\, 0 \leqslant x \leqslant 1} |w(t,x)| \to \min_z, \tag{46}$$

and which satisfies the restrictions

$$\frac{c_1 l^4}{EI} = z_1 \leqslant z(t) \leqslant z_2 = \frac{c_2 l^4}{EI}. \tag{47}$$

We confine ourselves to a particular case when

$$q = q_0 \sin \pi x \tag{48}$$

The general case is studied by employing a similar reasoning, but more cumbersome calculations are needed, since solutions of eqn (45) would be expended in series with respect to eigenfunctions of operator $\partial^4/\partial x^4$ with boundary conditions (40), and appropriate transformations would be accomplished for any term of the expansion.

For load (48), it is natural to seek the solution of eqn (45) in the form

$$w(t,x) = W(t) \sin \pi x, \tag{49}$$

where $W(t)$ is a function to be found. It is easy to show that function (49) satisfies the boundary conditions (40).

Substitution of expressions (48) and (49) into eqn (45) yields

$$\frac{d^2 W}{dt^2}+[\pi^4+z(t)]W(t) = q_0. \tag{50}$$

Introduce the new control $Z=(z-z_1)/(\pi^4+z_1)$. According to eqn (47), the function $Z(t)$ satisfies the inequality

$$0 \leqslant Z(t) \leqslant Z_1 = \frac{z_2-z_1}{\pi^4+z_1}. \tag{51}$$

Substituting expression for the function Z into eqn (50), we find

$$\frac{d^2 W}{dt^2}+(\pi^4+z_1)[1+Z(t)]W(t) = q_0. \tag{52}$$

Setting

$$W=\frac{q_0}{\pi^4+z_1}\tilde{W}, \quad t=\frac{1}{\sqrt{\pi^4+z_1}}\tilde{t},$$

we can rewrite eqn (52) as follows (for simplicity tildes are omitted):

$$\frac{d^2 W}{dt^2}+[1+Z(t)]W = 1. \tag{53}$$

The initial conditions for eqn (53) have the form

$$W(0)=0, \quad \frac{dW}{dt}(0)=0. \tag{54}$$

According to eqn (46), the problem of optimization consists in determining a function $Z^\circ(t)$ which fulfills restriction (51) and minimizes the functional

$$\max_{0 \leqslant t < \infty} |W(t)| \to \min_{Z(t)}. \tag{55}$$

Functional (55) is not strictly differentiable, and finding its minimum is not a simple problem. In order to bypass this difficulty we suggest to consider another problem with a sufficiently smooth functional, namely, the problem of minimizing the functional

$$\Phi(Z) = \int_0^T W^m(t)\,dt, \tag{56}$$

where T is a positive parameter, and m is a positive even integer. As is well-known, for $T \to \infty$ and $m \to \infty$, the solution of problem (56) tends (in a weak sense) to a solution of problem (55). Since the most important question for us is the minimal value of functional (55), this type of convergence is sufficient for our analysis.

Let us fix a perturbation ΔZ of the function Z and calculate the perturbation of the functional Φ caused by ΔZ. It follows from eqn (56) that

$$\Delta\Phi = m\int_0^T W_1^{m-1}(t)\Delta W_1(t)\,\mathrm{d}t, \tag{57}$$

where functions ΔW_1 and ΔW_2 satisfy the equations which follows from eqn (53)

$$\frac{\mathrm{d}\Delta W_1}{\mathrm{d}t} = \Delta W_2, \quad \frac{\mathrm{d}\Delta W_2}{\mathrm{d}t} = -(1+Z)\Delta W_1 - W_1\Delta Z, \tag{58}$$

with the initial conditions

$$\Delta W_1(0) = 0, \quad \Delta W_2(0) = 0. \tag{59}$$

Introduce dual variables ψ_1 and ψ_2 that satisfy the differential equations

$$\frac{\mathrm{d}\psi_1}{\mathrm{d}t} = (1+Z)\psi_2 + mW_1^{m-1}, \quad \frac{\mathrm{d}\psi_2}{\mathrm{d}t} = \psi_1 \tag{60}$$

with the boundary conditions

$$\psi_1(T) = 0, \quad \psi_2(T) = 0. \tag{61}$$

We multiply the first equality (58) by $(-\psi_1)$, the other equality (58) by ψ_2, and add to the functional (57). As a result we obtain

$$\Delta\Phi = \int_0^T \left\{ mW_1^{m-1}(t)\Delta W_1(t) - \psi_1\left(\frac{\Delta W_1}{\mathrm{d}t} - \Delta W_2\right) + \psi_2\left[\left(\frac{\Delta W_2}{\mathrm{d}t} + (1+Z)\Delta W_1 + W_1\Delta Z\right)\right]\right\}\mathrm{d}t$$

Integrating by parts and using the boundary conditions (59), (61) and eqns (60) we find

$$\Delta\Phi = \int_0^T \psi_2(t)W_1(t)\Delta Z(t)\,\mathrm{d}t. \tag{62}$$

Expression (62) together with the gradient method of minimization, see e.g., Polak (1971), allows the following iterative procedure to be applied for seeking the minimum. At the nth step of the iterative process we derive a control $Z^{(n)}(t)$. By using this control, we integrate eqn (53) and find $W_1^{(n)}(t) = W^{(n)}(t)$. By employing this function we integrate eqn (60) and obtain the function $\psi_2^{(n)}(t)$. Afterwards, we choose a positive constant α and construct a new control

$$Z^{(n+1)}(t) = \begin{cases} Z_1, & Z_n(t) - \alpha\Psi_n(t) > Z_1, \\ Z_n(t) - \alpha\Psi_n(t), & 0 < Z_n(t) - \alpha\Psi_n(t) < Z_1, \\ 0, & Z_n(t) - \alpha\Psi_n(t) < 0, \end{cases} \tag{63}$$

where $\Psi_n = \psi_2^{(n)} W_1^{(n)}$. We calculate the value of the functional $\Phi(Z^{(n+1)})$. If this value is smaller than $\Phi(Z^{(n)})$, we treat the control $Z^{(n+1)}$ as a new control and proceed calculations at the next step of iterations. If $\Phi(Z^{(n+1)}) \geqslant \Phi(Z^{(n)})$, we reduce α value twice and repeat the calculations of $Z^{(n+1)}$. According to eqn (62), after some iterations the inequality $\Phi(Z^{(n+1)}) < \Phi(Z^{(n)})$ will be fulfilled.

By using this algorithm, we calculate the control $Z(t) = Z^{(N)}(t)$ which depends on four parameters: N, the number of iterations; T, the length of time interval; m, the parameter of exponent; and Z_1, the limitation on the control. Only the latter parameter has a physical meaning, the others were introduced to perform the numerical procedure.

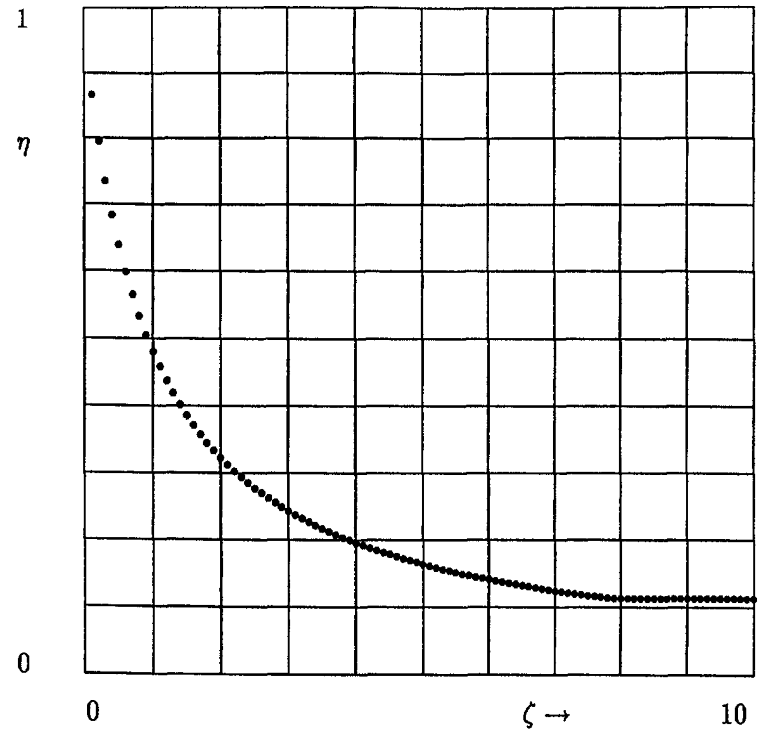

Fig. 3. The ratio η of the maximal deflection for a controllable beam to the maximal deflection for an uncontrollable beam vs the limitation ζ on the controllable rigidity of the foundation.

To study the influence of these parameters on the optimal control, we carry out numerical simulation of the algorithm. The results of numerical analysis show that for $N \geqslant 80$, neither the final control Z, nor the final value of the functional Φ depend on the number of iterations. Thus, setting $N = 80$, we can exclude this parameter from our consideration.

For $T \geqslant 2$, the dependence of the functional Φ on T becomes extremely feeble. For example, for $m = 2$ and $Z_1 = 1$, the minimal value of Φ equals 0.978 for $T = 2$, 1.035 for $T = 12$ and 1.059 for $T = 20$. This means that we can set $T = 20$ and treat this value as the numerical "infinity". The dimensionless parameter T determines time in units which equal the maximal period of natural oscillations for an appropriate elastic beam without foundation. The above calculation shows that it suffices to control the foundation rigidity only on the interval of time which equals twenty periods of natural oscillations for the corresponding non-controllable elastic beam.

The dependence of the functional Φ on the parameter m is also very weak. For example, for $T = 12$ and $Z_1 = 1$, the minimal value of Φ equals 1.035 for $m = 2$, 4 and 6. Therefore, we can set $m = 6$ and fix this value in numerical simulations.

After excluding these technical parameters, our attention is drawn to the dependence of $\max_{t \geqslant 0} |W(t)|$ on restriction Z_1. The dependence of the dimensionless parameter

$$\eta = \frac{\max_{t \geqslant 0} |W(t)|_{Z_1 = \zeta}}{\max_{t \geqslant 0} |W(t)|_{Z_1 = 0}}$$

on ζ is essential, and it is plotted in Fig. 3. The results show that the parameter η decreases monotonously in ζ and reaches its minimal value for $\zeta \approx 8$. This leads to two important conclusions: (i) by using the optimal control of the intelligent foundation rigidity we can reduce the maximal deflection of the beam by about 8 times ($\eta|_{\zeta=0}/\eta|_{\zeta=8} = 8.3$); (ii) to achieve the maximal effect of damping, it is not necessary to increase the foundation rigidity *ad infinity*, it is sufficient to increase the initial rigidity by only 8 times with the same efficiency.

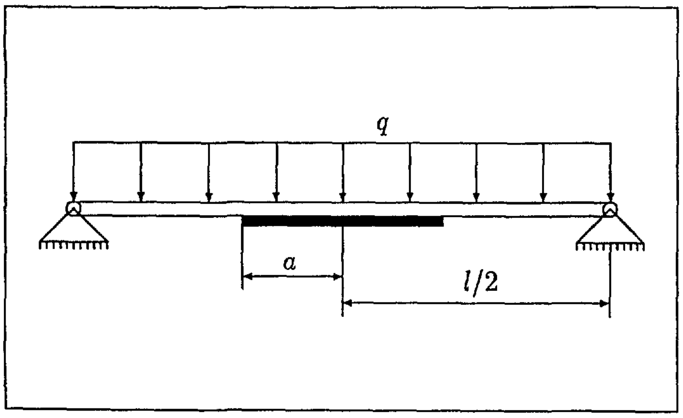

Fig. 4. An intelligent, simply supported beam with an actuator.

It is of interest to study what kind of control ensures the miminum of the maximal deflection. The numerical results show that optimal control Z^0 is a piece-wise constant function with two switch points. First, the foundation rigidity takes its maximal admissible value Z_1. Close to the moment t_1 when the beam deflection is maximal, the optimal rigidity becomes minimal, and remains minimal during practically the whole return movement of the beam. Close to moment t_2 when the beam deflection is maximal and directed oppositely to the foundation, the rigidity becomes maximal again, and keeps this value without changes until the end of the process.

5. OPTIMIZATION OF GEOMETRY OF AN INTELLIGENT STRUCTURE

Let us consider a simply supported elastic beam with the length l linked with an actuator. The actuator is modeled as an elastic beam with length $2a$, which is perfectly bonded to the beam and is located symmetrically with respect to the beam center, see Fig. 4.

At instant $t = 0$, distributed transverse load with a constant intensity q is applied to the beam, and an electrical signal is transmitted to the piezoelectrical actuator. This signal produces a residual compressive strain ε_a in the actuator. The beam deforms under the action of external load and actuator's compression. The bending moment M relates to the beam deflection w through the expression

$$M = \begin{cases} A\dfrac{d^2w}{dx^2}, & 0 \leqslant x < \dfrac{l}{2} - a, \\ A\dfrac{d^2w}{dx^2} - B\varepsilon_a, & \dfrac{l}{2} - a \leqslant x \leqslant \dfrac{l}{2} + a, \\ A\dfrac{d^2w}{dx^2}, & \dfrac{l}{2} + a < x \leqslant l, \end{cases} \tag{64}$$

where parameters A and B are determined by eqn (16). The sign of the right-hand side of eqn (64) differs from the sign in the right-hand side of eqn (14) because we treat here the compressive residual strain ε_a as positive.

The equilibrium equation is written as follows:

$$\frac{d^2M}{dx^2} = q. \tag{65}$$

Integration of eqn (65) with the boundary conditions $M(0) = M(l) = 0$ implies that

$$M(x) = \int_0^l G(x,\xi) q \, d\xi = -\frac{q}{2} x(l-x), \tag{66}$$

where

$$G(x,\xi) = \begin{cases} \xi\left(\frac{x}{l}-1\right) & \xi < x, \\ x\left(\frac{\xi}{l}-1\right) & \xi \geqslant x \end{cases} \tag{67}$$

is the Green function for an appropriate boundary problem.

Substitution of expression (64) into eqn (66) yields

$$\frac{d^2 w}{dx^2} = \begin{cases} -\frac{q}{2A} x(l-x), & 0 \leqslant x < \frac{l}{2} - a, \quad \frac{l}{2} + a < x \leqslant l, \\ \frac{1}{A}\left[B\varepsilon_a - \frac{q}{2} x(l-x)\right], & \frac{l}{2} - a \leqslant x \leqslant \frac{l}{2} + a. \end{cases}$$

Integration of these equations with the boundary conditions $w(0) = w(l) = 0$ yields

$$w(x) = \frac{1}{A}\left[B\varepsilon_a \int_{(l/2)-a}^{(l/2)+a} G(x,\xi) d\xi - \frac{q}{2} \int_0^l G(x,\xi)\xi(l-\xi) \, d\xi\right]. \tag{68}$$

Calculating the integrals in eqn (68) we obtain

$$w(x) = \frac{1}{A}\left[\frac{q}{24} x(l-x)(l^2 + lx - x^2) - B\varepsilon_a \Psi(x)\right], \tag{69}$$

where

$$\Psi(x) = \begin{cases} ax, & 0 \leqslant x < \frac{l}{2} - a, \\ \frac{1}{2}\left[x(l-x) - \left(\frac{l}{2} - a\right)^2\right], & \frac{l}{2} - a \leqslant x < \frac{l}{2} + a, \\ a(l-x), & \frac{l}{2} + a < x \leqslant l. \end{cases}$$

Introduce the dimensionless variables

$$x_* = \frac{x}{l}, \quad a_* = \frac{a}{l}, \quad \varepsilon_* = \frac{24 B\varepsilon_a}{ql^2}.$$

In the new notation, eqn (69) can be written in the form:

$$w(x) = \frac{ql^4}{24A}[x_*(l-x_*)(1 + x_* - x_*^2) - \varepsilon_{a*}\Psi_*(x_*)], \tag{70}$$

where

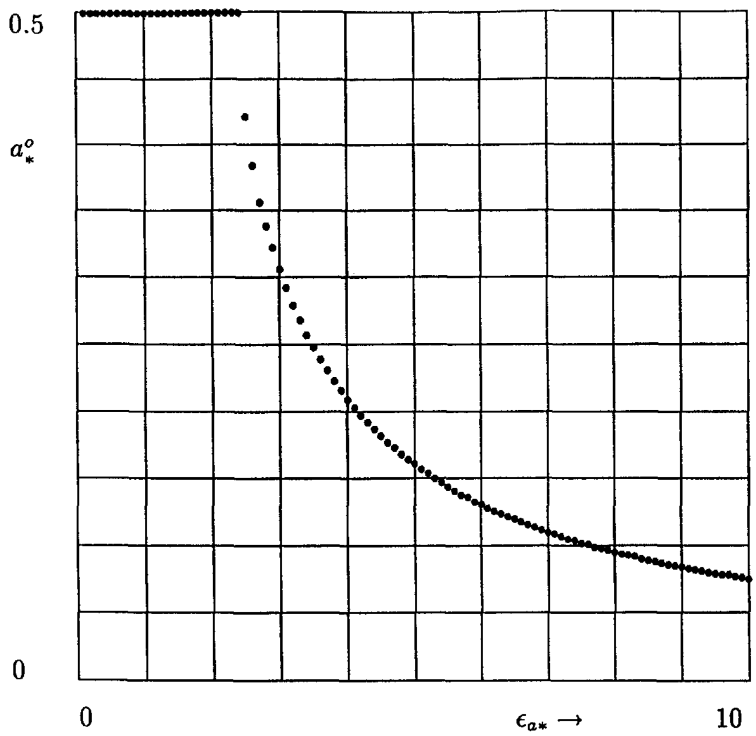

Fig. 5. The optimal dimensionless length of the actuator a^0_* vs the dimensionless residual strain ε_{a*}.

$$\Psi_*(x_*) = \begin{cases} a_* x_*, & 0 \leqslant x_* < \frac{1}{2} - a_*, \\ \frac{1}{2}[x_*(l - x_*) - (\frac{1}{2} - a_*)^2], & \frac{1}{2} - a_* \leqslant x_* \leqslant \frac{1}{2} + a_*, \\ a_*(l - x_*), & \frac{1}{2} + a_* < x_* \leqslant 1. \end{cases}$$

The optimization problem is formulated as follows : for a given intensity of the residual strain ε_{a_*} find parameter a^0_* which minimizes the maximal deflection

$$\max_{0 \leqslant x \leqslant l} |w(x)| \rightarrow \min_{a_*}, \tag{71}$$

and which satisfies the natural restriction $0 \leqslant a_* \leqslant \frac{1}{2}$.

Problem (70) and (71) is rather simple from the mathematical standpoint, and does not require special methods for its solving. Thus, the dependence of the optimal length of the actuator a_* on the residual strain ε_{a*} can be derived numerically. A surprising result is obtained when we plot this dependence, see Fig. 5. According to the numerical analysis, a long actuator with the length equal to the length of the beam is optimal only for relatively small residual strains, $\varepsilon_{a*} \leqslant \varepsilon^0_{a*} \approx 2.5$. For $\varepsilon_{a*} > \varepsilon^0_{a*}$, the optimal length of the actuator diminishes with an increase in ε_{a*} and tends to zero when $\varepsilon_{a*} \rightarrow \infty$. Returning to the dimensional variables, we find the following formula for the critical value of the strain :

$$\varepsilon^0_a = 0.21 \frac{ql^2(Eh + E_a h_a)}{EE_a h h_a (h + h_a)}.$$

For $\varepsilon_a > \varepsilon^0_a$, the optimal length of the actuator is less than the beam length, and the optimal length can be found by using the plot in Fig. 5.

6. CONCLUSIONS

The paper is concerned with basic aspects of a new theory of intelligent composite structures. We derive the governing equations, formulate the major optimization problems in the theory of intelligent structures, and discuss similarities and discrepancies between the theory of intelligent structures and the theory of optimal control.

The basic optimization problems are illustrated by three examples in which we emphasize three major sources of control for an intelligent composite structure with sensors and actuators, namely, residual strains, material properties, and the geometry of the structure.

In the first, example, we derive an explicit expression for the optimal residual stress in an actuator which provides the minimal deflection of a cantilevered beam under static loading. It is shown that the actuator can reduce the maximal deflection by 28 times compared with the same beam without active control. We analyze the robustness of the obtained solution.

The second example is concerned with the optimal control of the foundation rigidity in the problem of vibrations damping for a simply supported beam under dynamic loading. We obtain an explicit expression for the optimal control as a piece-wise constant function with two switch points which are determined numerically. It is shown that the only dimensionless parameter of damping is the ratio of the maximal rigidity of the controllable Winkler foundation to its minimal rigidity. The maximal deflection decreases with the growth of this ratio and tends to its limiting value when the ratio becomes sufficiently large. The ratio determines the ultimate efficiency of damping: by using the controllable foundation we can reduce the maximal deflection by about 8 times.

The third example deals with the optimal design of an actuator for a two-layered, controllable, simply supported beam. The objective of the control is to reduce the maximal deflection by applying to the actuator a constant residual strain. It is shown that when this strain is rather small, less than the determined critical value ε_a^0, the optimal length of the actuator has to be maximal and equal to the length of the beam. However, for the strains, which exceed this critical value, the optimal length of the actuator is smaller than the length of the beam, and it diminishes up to zero with the growth of the applied strain.

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